[11:00-11:10] Review of tune-up exercise and review from last lecture

Plotting a sinusoid in MATLAB may not produce a smooth plot. Increasing the sampling rate can help. When the sampling rates are low, discrete-time plots of sinusoids can be hard to recognize as sinusoids.

A time shift of a sinusoid results in a phase shift. We can calculate the phase shift from the time shift exactly, and we can use phase unwrapping algorithms to estimate the time shift from the phase shift.

The time shift/phase shift of a wave propagating through space can be used for navigation and ranging, including radio navigation and ranging (RADAR), and sound navigation and ranging (SONAR).

[11:10-11:20] Sound frequency and wavelength

The lowest audible sound is about 20 Hz. However, it is very difficult to produce a pure 20 Hz sinusoid, since it would require a large and powerful subwoofer.

Wavelength (λ, in meters) is related to frequency (f, in Hertz) by the speed of the wave (c, in meters per second):

$$\lambda = \frac{c}{f}$$

Speed of sound in air: 340 m/s

Speed of sound in water: 1500 m/s

Speed of light/radio waves: 3×10^8

To produce larger wavelengths, you need loudspeaker or antenna whose size is proportional to the wavelength. Example:

$$\frac{340\ \mathrm{m/s}}{20\ \mathrm{Hz}} = 17\mathrm{m}$$

To reproduce a sampled signal, we require the sampling rate $f_s > 2f_{\max} \rightarrow f_{\max} < \frac{1}{2} f_s$

[11:20-] Sampling

Many signals are initially measured as continuous signals (e.g. sound converted from continuous pressure signal to a continuous voltage signal via transducer)

Sampling a continuous signal at equally spaced intervals T_n in time produces a discrete time signal:

$$\underbrace{y[n]}_{\text{discrete time signal}} = \underbrace{y(nT_s)}_{\substack{\text{continuous signal evaluated at integer} \\ \text{multiples of the sampling period}}}$$

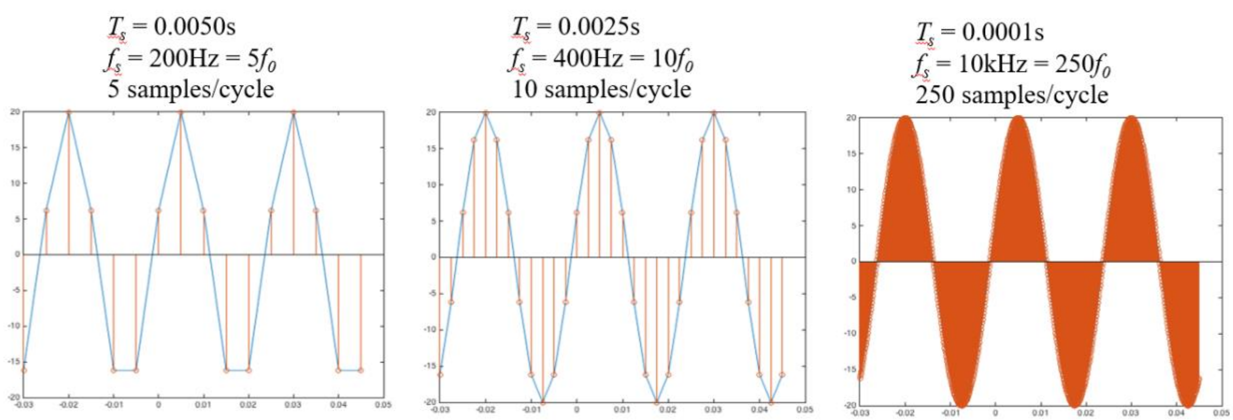
Example: sampling a sinusoid:

$$y(t) = A\cos(2\pi f_0 t + \phi)$$

$$y[n] = y(t)|_{t=nT_s} = A\cos\left(\underbrace{2\pi f_0 T_s}_{\widehat{\omega}_0} n + \phi\right)$$

$\widehat{\omega}_0 = 2\pi f_0 T_s = \frac{2\pi f_0}{fs}$ is the discrete time frequency in units of radians per sample.

How small does T_s need to be (or how high does f_s need to be) to for an accurate plot?



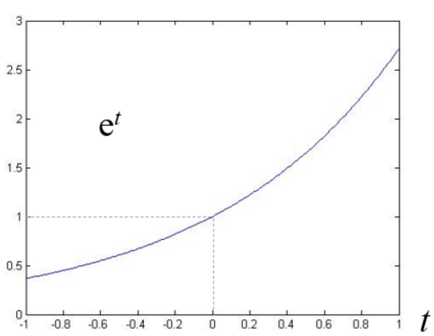
[11:35] Exponential signals

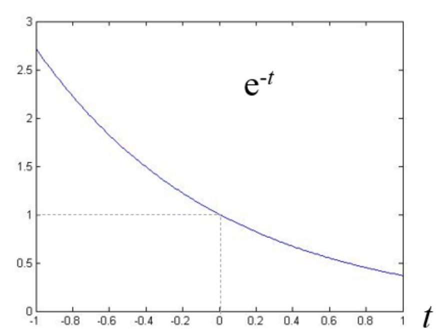
Real-valued exponential signals

Amplitude values are always non-negative

Might decay or not as t goes to infinity

$$\lim_{t\to\infty} e^{at} = \begin{cases} 0 & if\ a < 0 \\ 1 & if\ a = 0 \\ \infty & if\ a > 0 \end{cases}$$






```
t = -1 : 0.01 : 1;
e1 = exp(t);
plot(t, e1)
```

```
t = -1 : 0.01 : 1;
e2 = exp(-t);
plot(t, e2)
```

Complex exponentials are related to sinusoids:

$$\begin{aligned} e^{j\theta} &= \cos(\theta) + j\sin(\theta) \\ e^{-j\theta} &= \cos(\theta) - j\sin(\theta) \end{aligned} \rightarrow \begin{aligned} \cos\theta &= \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \\ \sin\theta &= \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \end{aligned}$$

[11:45-12:15] Signal properties and spectrum representation

Signals can be defined or described in many ways:

- As a function (e.g. $\cos(t)$)
- As a sequence of numbers
- By its properties (e.g. symmetric)
- As a piecewise combination of other signals

Signals can be represented spectrally as a combination of sinusoids:

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) = X_0 + \mathrm{Re}\left\{\sum_{k=1}^{N} X_k e^{j2\pi f_k t}\right\}$$

Where A_0 is a real-valued and $X_0 = A_0$ and $X_k = A_k e^{j\phi_k}$

Example (slide 1-16)

$$x(t) = \cos(2\pi \cdot 440 \cdot t)$$

$$x(t) = \cos(A_0) + A_1\cos(2\pi \cdot 440 \cdot t) + A_1\cos(2\pi \cdot 880 \cdot t)$$

A_0	0		
A_1	1	A_2	0
f_1	440 Hz	f_2	880 Hz
ϕ_1	0	ϕ_2	0

$$y(t) = x^2(t) = \cos^2(2\pi \cdot 440 \cdot t) = \frac{1}{2} + \frac{1}{2}\cos(2\pi \cdot 880 \cdot t)$$

A_0	½		
A_1	1	A_2	0.5
f_1	440 Hz	f_2	880 Hz
ϕ_1	0	ϕ_2	0

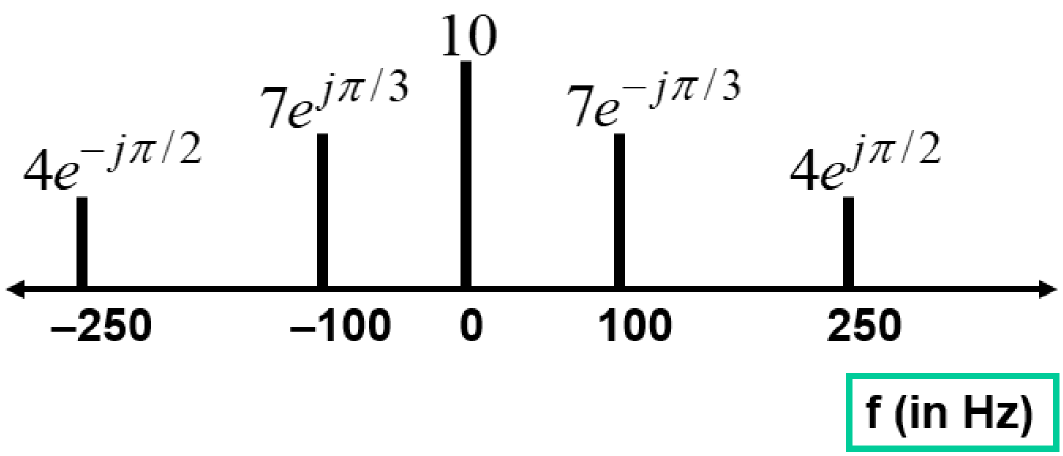
Two sided spectrum:

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) = X_0 + \sum_{k=1}^{N} \left\{ \frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t} \right\}$$

Where $x_k = A_k e^{j\phi_k}$ and $X_k^* = A_k e^{-j\phi_k}$

Example:

$$\begin{aligned} x(t) &= 10 + 14\cos\left(200\pi t - \frac{\pi}{3}\right) + 8\cos\left(500\pi t - \frac{\pi}{2}\right) \\ &= 10 + 7e^{\frac{-j\pi}{3}} e^{j2\pi 100t} + 7e^{\frac{j\pi}{3}} e^{-j2\pi 100t} + 4e^{\frac{j\pi}{2}} e^{j2\pi 250t} + 4e^{\frac{-j\pi}{2}} e^{-j2\pi 250t} \end{aligned}$$



[12:15-] Beat Notes

When two sinusoids are multiplied, the result is two frequencies (sum and difference)

$$\begin{aligned} x(t) &= \cos(2\pi 10t)\cos(2\pi 1000t) \\ &= \frac{1}{2}\cos\left(2\pi\, 1010\, t - \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2\pi\, 990\, t - \frac{\pi}{2}\right) \end{aligned}$$

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Wavelength $\lambda = \frac{c}{f}$ ← [m/s]

[m] ← [Hz]

c is the speed of propagation in the medium

speed of sound in air: $c = 340$ m/s

speed of sound in water: $c = 1500$ m/s

speed of light: $c = 3 \times 10^8$ m/s

speed of RF propagation: $c = 3 \times 10^8$ m/s

for 20 Hz acoustic signal, $\lambda = \frac{340 \text{ m/s}}{20 \text{ Hz}}$

$\lambda = 17$ m

Sampling – Slide 2-5

$f_s > 2 f_{max} \rightsquigarrow f_{max} < \frac{1}{2} f_s$

$y(t) = A \cos(2\pi f_0 t + \phi)$

Sample at $t = n T_s$,

$y[n] = y(t)\big|_{t = nT_s} = A\cos(2\pi f_0 (n T_s) + \phi)$

$= A \cos(2\pi f_0 T_s n + \phi)$

$\hat{\omega}_0 = 2\pi f_0 T_s = 2\pi \frac{f_0}{f_s}$

[rad/sample] ← $\hat{\omega}_0$ discrete-time frequency